

Find $\frac{d}{dx} \frac{\tanh^{-1}(x^3)}{x^2}$. Simplify your final answer as a single fraction.

SCORE: ____ / 4 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$\textcircled{1} \left(\frac{1}{1-(x^3)^2} \cdot 3x^2 \right) x^2 - 2x \tanh^{-1} x^3 \quad \textcircled{1}$$

$$\frac{(x^2)^2}{x^4} \quad \textcircled{1}$$

$$= \frac{3x^4 - 2(1-x^6)\tanh^{-1} x^3}{x^4}$$

$$= \frac{3x^3 - 2(1-x^6)\tanh^{-1} x^3}{x^3(1-x^6)} \quad \textcircled{1}$$

If $\tanh x = -\frac{2}{3}$, find $\sinh x$.

SCORE: ____ / 4 PTS

$$1 - \tanh^2 x = \operatorname{sech}^2 x \quad (1)$$

$$1 - \frac{4}{9} = \operatorname{sech}^2 x$$

$$\frac{5}{9} = \operatorname{sech}^2 x$$

$$\operatorname{sech} x = \frac{\sqrt{5}}{3} \quad (1)$$

SINCE $\operatorname{sech} x > 0$

FOR ALL x

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5} \quad (1)$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$-\frac{2}{3} = \frac{\sinh x}{\frac{3\sqrt{5}}{5}} \quad (1)$$

$$\sinh x = -\frac{2}{3} \cdot \frac{3\sqrt{5}}{5} = -\frac{2\sqrt{5}}{5} \quad (1)$$

EITHER IS
OK

Find $\lim_{x \rightarrow -\infty} \tanh x$ algebraically.

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$$\lim_{x \rightarrow -\infty} \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{0 - 1}{0 + 1} = -1$$

$\lim_{x \rightarrow -\infty} \frac{e^{2x} - 1}{e^{2x} + 1}$ $0 - 1$ $0 + 1$ -1

$\left(\frac{1}{2}\right)$ $\left(\frac{1}{2}\right)$ (1)

For this question, you may use the formulae for $\frac{d}{dx} \sinh x$ and/or $\frac{d}{dx} \cosh x$ without proving them.

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If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

- [a] Without using the logarithmic formula for $\sinh^{-1} x$, prove the formula for $\frac{d}{dx} \sinh^{-1} x$.

$$y = \sinh^{-1} x$$

$$\sinh y = x \quad (1)$$

$$(\cosh y) \frac{dy}{dx} = 1 \quad (1)$$

$$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1+x^2}} \quad (1) \quad (1)$$

$$\cosh^2 y - \sinh^2 y = 1 \quad (1)$$

$$\cosh y = \sqrt{1 + \sinh^2 y}$$

SINCE $\cosh y > 0$
FOR ALL y

- [b] Without using the exponential formula for $\operatorname{sech} x$, prove the formula for $\frac{d}{dx} \operatorname{sech} x$.

$$\frac{d}{dx} \frac{1}{\cosh x} = \frac{0 \cdot \cosh x - 1 \cdot \sinh x}{\cosh^2 x} = \frac{-\sinh x}{\cosh^2 x} = -\operatorname{sech} x \tanh x$$

OR (1) (1)

$$\frac{d}{dx} (\cosh x)^{-1} = -(\cosh x)^{-2} \sinh x = -\operatorname{sech} x \tanh x$$

Prove the logarithmic formula for $\tanh^{-1} x$ given in your textbook.

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NOTE: This is NOT a question about derivatives.

$$y = \tanh^{-1} x$$

$$\tanh y = x$$

$$\frac{e^{2y} - 1}{e^{2y} + 1} = x \quad (1)$$

$$e^{2y} - 1 = x e^{2y} + x \quad (1)$$

$$e^{2y} - x e^{2y} = 1 + x$$

$$(1 - x) e^{2y} = 1 + x \quad (1/2)$$

$$e^{2y} = \frac{1+x}{1-x}$$

$$2y = \ln \frac{1+x}{1-x} \quad (1)$$

$$y = \frac{1}{2} \ln \frac{1+x}{1-x} = \tanh^{-1} x$$